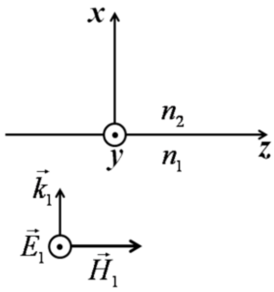

OC Problem Set 3

Friday, May 15, 2015.

Problem 1: Incident, transmitted, reflected, and standing TE-waves

A monochromatic, plane TE-wave $\vec{E}_1$ as depicted on the right-hand side is incident *perpendicularly* onto the boundary plane $x = 0$ which separates medium 1 ($x < 0$) from medium 2 ($x > 0$). Both media are non-magnetic and non-conducting. The dielectric constants of medium 1 and medium 2 are n_1 and n_2 respectively.

a) Which Cartesian field components make up the incident TE-wave $\vec{E}_1$? Which components make up the propagation vector $\vec{k}_1$ of $\vec{E}_1$?
Write down the total electric field in medium 1 using complex notation! Observe that it is a superposition of incident wave $\vec{E}_1$ and reflected wave $\vec{E}_3$.
What is the relation between $\vec{k}_3$ of the reflected and $\vec{k}_1$ of the incident wave?
Is the superposition of $\vec{E}_1$ and $\vec{E}_3$ a standing wave, or a propagating wave, or what else?
Write down the electric field $\vec{E}_2$ of the transmitted wave in complex notation. What type of wave is it?

b) The relation between the electric field $\vec{E}_s$ and the magnetic field $\vec{H}_s$ ($s = 1, 2, 3$) will be determined next:
Apply Maxwell's equation $\mathrm{curl}\vec{E}_s = -\mathrm{j}\omega\mu_0\vec{H}_s$ to the electric field $\vec{E}_s$ of part a) and show that with the transverse field components the transverse wave impedance Z_s can be defined: $H_s = E_s / Z_s$.
Write Z_1, Z_2, and Z_3 as a function of n_1 and n_2. **Hint:** $k = nk_0 = n\dfrac{\omega}{c}$.

In the following, the amplitude reflection coefficient r_E of the electric field at the boundary $x = 0$ will be determined for the case of perpendicular incidence as a function of the transverse wave impedances Z_1 and Z_2, or the refractive indices n_1 and n_2, respectively. The reflection coefficient r_E is defined as the amplitude ratio of the reflected wave and the incident wave: $r_E = E_3 / E_1$.

c) Formulate the boundary conditions at $x = 0$ for the electric and magnetic field components.

d) Use the boundary conditions of part c) to determine the amplitude reflection coefficient r_E as a function of Z_1 and Z_2, or as a function of n_1 and n_2, respectively.

e) Which reflection coefficients result for the cases $n_2 \gg n_1$ and $n_2 \ll n_1$? For both cases, plot the amplitude of the electric field in medium 1 ($x < 0$). **Hint:** Take the real part of the superposition of incident wave $\vec{E}_1$ and reflected wave $\vec{E}_3$ from part a).

f) For those of you interested in transmission line theory, How would you call these cases in terms of a transmission line (short circuit / open circuit)? (This part is optional)

For questions and suggestions on the OC tutorial please contact:

Stefan Wolf, Bldg. 30.10, Room 1.23, Phone: 0721/608-47173, s.wolf@kit.edu
Wladick Hartman, Room 2.23, Phone: 0721/608-48954, wladislaw.hartmann@kit.edu